Flux Tube Model Signals for Baryon Correlations in Heavy Ion Collisions

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Abstract

The flux tube model offers a pictorial description of what happens during the deconfinement phase transition in QCD. The 3-point vertices of a flux tube network lead to formation of baryons upon hadronisation. Therefore, correlations in the baryon number distribution at the last scattering surface are related to the preceding pattern of the flux tube vertices, and provide a signature of the nearby deconfinement phase transition. I discuss the nature of the expected signal, which should be observable in heavy ion collisions at RHIC and LHC.
Heavy Ion Collisions

STAR event at RHIC
Au+Au collision at 100+100 GeV/nucleon

ALICE event at LHC
Pb+Pb collision at 1.38+1.38 TeV/nucleon

Thousands of hadrons are produced.
Only charged ones are detected.
Transverse coverage: $\theta_m < \theta < \pi - \theta_m$
CMBR Observations

The most precisely measured black body spectrum in nature

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WMAP(2010) temperature anisotropy data
\[ \Delta T \sim 10^{-5}T \]
Temperature correlations (scalar) arise from the density fluctuations at the last scattering surface.

The data are accurate, and fit well to predictions of inflationary models.
CMBR Angular Correlations

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Polarisation correlations (tensor) arise from scattering in the plasma at the last scattering surface.

The data are poor.
Fireball Evolution Stages

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(5) Elastic and resonant scattering (mediated largely by pions) ceases, with kinetic freeze-out at $T_{\text{kin}} \simeq 120\text{MeV}$. 
Experimental Signals

Multiplicities and distributions of various particles are detected. Only charged hadrons observed in sufficiently transverse directions (to avoid the unscattered beams).

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(a) Initial approach to equilibrium signals: Direct photons and leptons, heavy quark jets, high $p_T$ jets, the elliptic flow.

(b) Quasi-equilibrated QGP signals: Moderate $p_T$ hadrons produced close to the fireball surface.
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Energy-momentum distributions and particle abundances provide information about the temperature (thermalised hadron resonance gas models).

Angular distributions can see through the scatterings to the correlation patterns in the QGP (assuming low diffusion).
Schematic description of the phase structure of QCD in the $m - T - \mu$ space. First order transition surfaces are shown shaded, and critical lines are shown dotted. Colour superconductor phases occurring at large chemical potential are omitted.
QCD Phase Transitions

(1) \( m = \infty, \ N \geq 3 \): First order finite temperature deconfinement transition, governed by the breaking of the global \( Z_N \) centre symmetry of the Polyakov loop.

(2) \( m = 0 = \mu, \ N_f \geq 3 \): First order finite temperature chiral transition, governed by the restoration of the flavour \( SU(N_f)_V \) symmetry to \( SU(N_f)_L \otimes SU(N_f)_R \).

(3) \( m = 0 = T, \ \mu \simeq \text{constituent quark mass} \): First order baryon condensation phase transition, where the vacuum structure changes from \( \langle \bar{\psi}\psi \rangle \neq 0 \) to \( \langle \psi^\dagger\psi \rangle \neq 0 \).

First order phase transitions are stable against small changes of symmetry breaking perturbations. The above three transitions extend inward, to varying extent, from the boundaries of the phase structure.

No phase transition for the physical values of the quark masses (unless \( \mu \) is sufficiently large). But the three nearby transitions have their imprints in the cross-over region.
Flux Tubes in QCD

QCD exhibits dual superconductivity with linearly confined colour-electric flux. (Nambu, ’t Hooft, Mandelstam)

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Meson and baryon wavefunctions are represented by the invariant tensors \( \delta_{ab} \) and \( \epsilon_{abc} \). Other multi-quark hadrons (except for nuclei) are phenomenologically not prominent.

Possible flux tube configurations connecting a static quark-antiquark pair, as the temperature is increased (from top to bottom), and when baryonic vertices are included (from left to right).
A colour-electric flux tube can break when dynamical quarks are included in the theory.
(a) A flux tube produced by static colour sources.
(b) Its breaking by a quark-antiquark pair appearing from the vacuum.
(c) Its breaking by a baryon appearing from the vacuum at finite chemical potential.
**Flux Tube Model Variables**

The link and site variables for the flux tube model.

**Energy:** \( E = \sigma a \sum_{i,\mu} |n_{i,\mu}| + m \sum_{i,f} |p_{i,f}| + v \sum_i |q_i| \)

**Gauss’s Law:** \( \sum_{\mu} (n_{i,\mu} - n_{i-\mu,\mu}) - \sum_f p_{i,f} + Nq_i \equiv \alpha_i = 0 \)

**Baryon Number:** \( B = \frac{1}{N} \sum_{i,f} p_{i,f} = \sum_i q_i \)
Grand Canonical Partition Function

\[ Z[T, \mu] = \sum_{n_i, \mu, p_i, q_i} \exp \left[ -\frac{1}{T}(E - \mu NB) \right] \prod_i \delta_{\alpha_i, 0} \]

The constraint can be solved by changing to dual variables:

\[ \delta_{\alpha_i, 0} = \int_{-\pi}^{\pi} \frac{d\theta_i}{2\pi} e^{i\alpha_i \theta_i} \]

Sum over \( n_i, \mu, p_i, q_i \) can then be explicitly carried out:

\[
Z[T, \mu] = \int_{-\pi}^{\pi} \prod_i \frac{d\theta_i}{2\pi} \prod_{i, \mu} (1 + 2e^{-\sigma a/T} \cos(\theta_i + \mu - \theta_i)) \times \]
\[
\times \prod_i \left( 1 + 2e^{-m/T} \cos \left( \theta_i + i\mu \frac{\mu}{T} \right) \right)^{2N_f} \prod_i (1 + 2e^{-v/T} \cos(N\theta_i))
\]
Phenomenological Features

The model is in the universality class of the XY spin model, with an ordinary and a $Z(N)$ symmetric magnetic field.

\[
Z[T, \mu] = \int_{-\pi}^{\pi} \prod_i \frac{d\theta_i}{2\pi} \exp \left[ J \sum_{i, \mu} \cos(\theta_{i+\mu} - \theta_i) + h \sum_i \cos \left( \theta_i + i\frac{\mu}{T} \right) + p \sum_i \cos(N\theta_i) \right]
\]

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J \simeq 2e^{-\sigma a/T}, \quad h \simeq 4N_f e^{-m/T}, \quad p \simeq 2e^{-v/T}
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Introduction of a static quark source at site $j$ modifies the Gauss’s law constraint there as $\delta_{\alpha j,0} \rightarrow \delta_{\alpha j,-1}$.

Its free energy is given by $\exp(-F_q/T) = \langle \exp(-i\theta_j) \rangle$.

$\theta_i$ corresponds to the phase of the Polyakov loop. Flux tube and Polyakov loop descriptions of deconfinement in finite temperature gauge theory are dual to each other.
Baryon Number Correlations

Focus on the position space picture of the flux tube network.

In every flux tube cluster, any neighbour of a vertex is an anti-vertex and vice versa. Production and annihilation of vertices stops at chemical freeze-out. Thereafter, every vertex yields a baryon and every anti-vertex an antibaryon.
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Heavy ion collisions produce a sizeable number of antibaryons, from an initial state that has none, implying that the fireball produces a good number of (anti)vertices. Specific patterns in the distribution of these vertices can be searched for, using techniques similar to those used to analyse the temperature fluctuations in the CMBR.
Pair Distribution Function

Density: \[ \rho(\vec{r}) = \langle \sum_\alpha \delta(\vec{r} - \vec{r}_i) \rangle \]

Correlation: \[ \rho(\vec{r}) \, g(\vec{r}, \vec{r}') \, \rho(\vec{r}') = \langle \sum_{i \neq j} \delta(\vec{r} - \vec{r}_i) \, \delta(\vec{r}' - \vec{r}_j) \rangle \]

In homogeneous and isotropic fluids, \( \rho \) is independent of \( \vec{r} \) and \( g \) depends only on \( |\vec{r} - \vec{r}'| \), resulting in

\[ \rho \, g(r) = \langle \sum_{i \neq 0} \delta(\vec{r} - \vec{r}_i) \rangle \]
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For an ideal gas with no correlations, \( g(r) = 1 \).
For objects with hard core repulsion, \( g(0) = 0 \), and beyond the hard core \( g(r) \) tends to its asymptotic value exhibiting damped oscillations.
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For baryon number distributions, \( g_{|v|} \) is correlation insensitive reference function, while \( g_v \) (including vertex signs \( q_i q_j \)) is sensitive to vertex-antivertex correlations. The contrast between the two measures the correlations.
Schematic representation of the pair distribution functions $g_{|v|}(r)$ and $g_v(r)$. The former is similar to that for objects with hard core repulsion. The latter is for a percolating flux tube network where vertices and anti-vertices alternate (similar to charges in ionic liquids).
Angular Projection

Projection of 3-dimensional pair distribution function onto the surface of the fireball smears its oscillatory structure.

\[ w(\alpha) = \int_{r_{\text{min}}}^{r_{\text{max}}} S(\alpha, r) g(r) \, dr, \quad \int_0^\pi S(\alpha, r) \, d\alpha = 1 \]

\[ \cos \alpha = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi') \]
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\end{align*}
\]

When inter-vertex separation \( r \ll \) the fireball radius \( R \),

\[
S(\alpha, r) = \int_0^R \frac{3a^2 da}{R^3} \int_0^\pi \frac{\sin \beta \ d\beta}{2} \ \delta \left( \alpha - \frac{r}{a \sin \beta} \right)
\]

\[
= \begin{cases} 
9\pi r^3 / (16R^3 \alpha^4) & : \alpha \geq r / R \\
3r^3 (12\lambda - 8 \sin(2\lambda) + \sin(4\lambda)) / (32R^3 \alpha^4) & : \sin \lambda = \alpha R / r \leq 1
\end{cases}
\]

Internucleon separation in nuclear matter is \( \sim 2 \text{fm} \).
Fireball radius in central heavy ion collisions is \( \sim 6 \text{fm} \).
Data Parametrisation

The experimental set up has axial symmetry around the beam axis, and reflection (parity) symmetry $\theta \leftrightarrow \pi - \theta$.

$$b(\hat{n}) \equiv b(\theta, \phi) = \frac{1}{\sqrt{2\pi}} \sum_{\sigma = \pm} \sum_{m = -\infty}^{\infty} b_{m}^{\sigma}(\theta) \, e^{im\phi}, \quad \langle b(\hat{n}) \rangle = \frac{b_{0}^{+}(\theta)}{\sqrt{2\pi}}$$
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Two-point correlations:

$$\langle b(\hat{n})b(\hat{n}') \rangle = \frac{1}{2\pi} \sum_{\sigma=\pm} \sum_{m=-\infty}^{\infty} C_{m}^{\sigma}(\theta, \theta') \ e^{im(\phi-\phi')}$$

$$C_{m}^{\sigma}(\theta, \theta') = \frac{1}{2\pi} \int_{0}^{2\pi} d\phi \int_{0}^{2\phi} d\phi' \ e^{-im(\phi-\phi')} \langle b^{\sigma}(\hat{n})b^{\sigma}(\hat{n}') \rangle$$

$$[C_{m}^{\sigma}(\theta, \theta')]_{c} = C_{m}^{\sigma}(\theta, \theta') - \delta_{m0} \ \delta_{\sigma} \ b_{0}^{+}(\theta) \ b_{0}^{+}(\theta')$$

$C_{m}^{\sigma}(\theta, \theta')$ can be expanded in terms of Associated Legendre polynomials for homogeneous isotropic distributions.
Gaps Between Theory and Experiment

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Moreover, corrections need to be estimated for:
(1) Only approximate equilibration of the fireball,
(2) Non-uniformity of the QGP due to the elliptic flow,
(3) Baryon number diffusion after hadronisation.

Low diffusion and low viscosity are compatible because of high entropy of hadronic medium.
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Despite these gaps:
It is worthwhile to look for the two-point baryon number correlations in the experimental data, as a characteristic signature of the deconfinement phase transition, without worrying about accurate prediction of its magnitude.
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