Quantum Computation

Concepts and Prospects

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Theoretical foundation of the subject is clear.
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Theoretical foundation of the subject is clear.
Elementary hardware components work as predicted.
Large scale integration (say 10 or more components) is a technological challenge. Noone knows when that will arrive, or what a quantum computer will be used for.

It is inevitable

"Because the nature isn't classical, damn it . . ."

—R.P. Feynman

Laws of classical physics are convenient and useful,
and yet only approximations (that are not fully understood)
to the underlying laws of quantum physics.

Science: Observe and explain phenomena. Theorise!
Technology: Design and control phenomena. Optimise!

Yesterday's science becomes tomorrow's technology.

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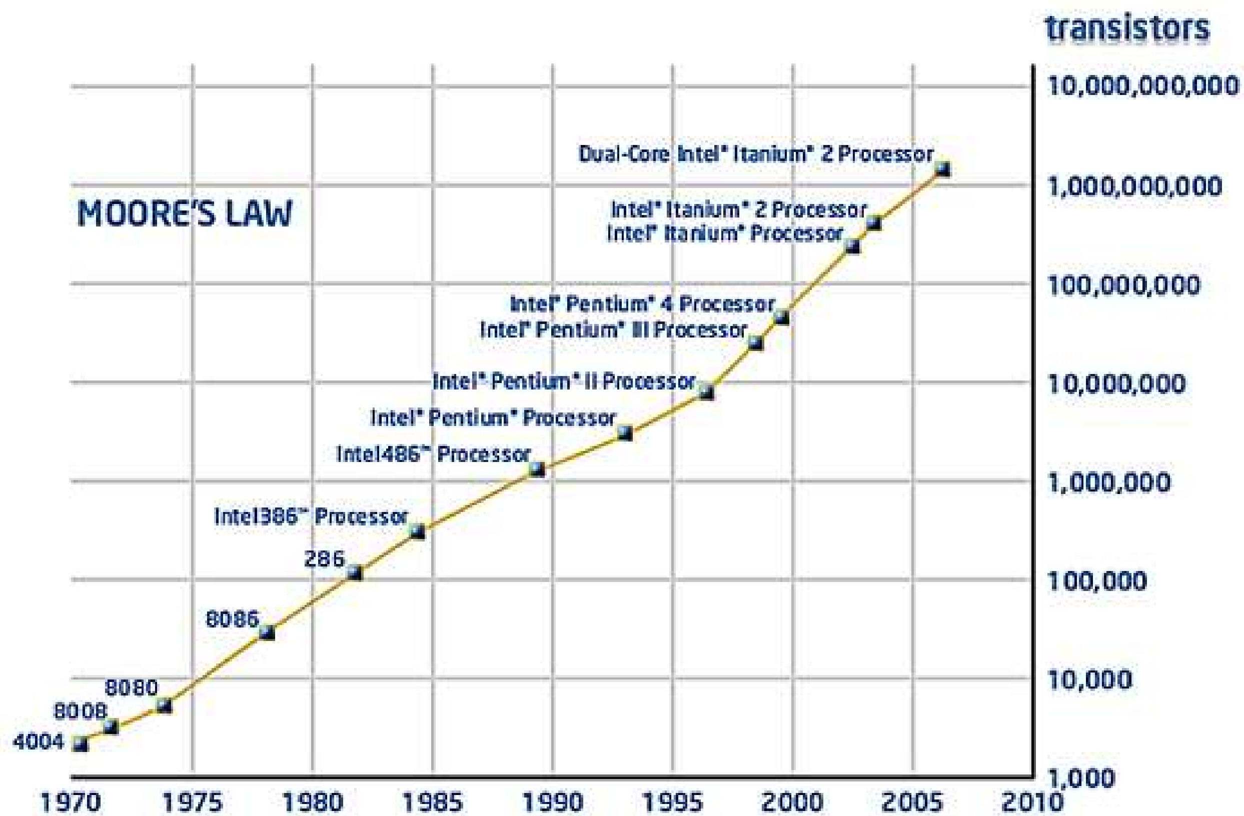
Quantum effects (discreteness, dispersion, tunnelling etc.) have been considered "undesirable nuisance" in the classical computer design.
Why not go to the other side, where classical effects (decoherence, thermal fluctuations etc.) become "undersirable nuisance" in the quantum computer design?

Shrinking computer circuits



Number of transistors on a chip doubles every two years.

1948: First transistor, size $\sim$ 1 cm. Today: VLSI circuits, size 45 nm.

Atomic size, 0.1 nm, is not very far!

(First nanotechnnology, and then decoherence, will have to be conquered along the way.)

It is a breakthrough

Computers are physical devices, not mere mathematical entities to implement algorithms. Quantum mechanics demonstrates that complex numbers are physical.
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Superposition allows multiple signals at the same point at the same time. All of them can be simultaneously processed, and any one of them can be selectively observed (e.g. radio or cell-phone transmissions). This offers an SIMD parallel computing paradigm with no extra hardware. Which algorithms can exploit this?

Explorations

Discrete variables:

Qubit	Technology
Electron spin	Crystal defects
Nuclear spin	Nuclear magnetic resonance
Photon polarisation	Quantum optics, cavity QED
Two-level atom	Ion traps, Quantum dots
Magnetic flux quantum	Superconducting circuits
Non-abelian anyon	?Spin chains?

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Range of opinion polls on availibility of quantum computers

10 years, 20 years, 50 years, . . . , never!

TO

We already have quantum computers!

Basics

The simplest quantum system is a qubit, with two basis vectors $|0\rangle$ and $|1\rangle$ (e.g. $|\uparrow\rangle$ and $|\downarrow\rangle$ for an electron spin).
A generic qubit state is a 2-dim complex unit vector.

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1.$$

A quantum register is an ordered string of n qubits.
It is a complex unit vector in the 2^n-dim Hilbert space.

$$|x\rangle = \sum_{i_1,i_2\ldots i_n=0}^{1} c_{i_1 i_2 \ldots i_n} |x_{i_1}\rangle|x_{i_2}\rangle \ldots |x_{i_n}\rangle, \sum_{i_1,i_2\ldots i_n=0}^{1} |c_{i_1 i_2 \ldots i_n}|^2 = 1.$$

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A generic instruction is a rotation of the quantum state vector in the Hilbert space. It is a unitary transformation that is deterministic and fully reversible.

A measurement is a projection. In the computational basis, it yields the state $|x_{i_1}\rangle|x_{i_2}\rangle \ldots |x_{i_n}\rangle$ with probability $|c_{i_1 i_2 \ldots i_n}|^2$.
This operation is probabilistic and irreversible.

Applications

Complexity of a quantum algorithm is decided by the trade-off between the number of evolution steps and the number of states that can be coherently superposed.

The gain may be exponential, polynomial or just marginal (factor of 2).

The input and output states of a quantum computer are always mapped to classical states, through a suitable choice of basis vectors.

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Pattern recognition: Clever superposition and interference can amplify the desired feature. The gain depends on the structure present in the data.

Shor's Factorisation Algorithm

Multiplication is easy, but no polynomial (in the number of digits) classical algorithm for factoring a number is known. Security of public key cryptography systems (e.g. RSA) relies on this fact.

The problem of factoring a number N can be reduced to finding the period of the function $f(x) = a^x \bmod N$. (a is chosen coprime to N, modular exponentiation is easy, number of possible remainders is limited.)

Period r: $f(0) = 1,\ f(1) = a, \ldots,\ f(r) = a^r \bmod N = 1.$

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Whenever r is even, $(a^{r/2} - 1)(a^{r/2} + 1) = 0 \bmod N$. So $(a^{r/2} - 1)$ and/or $(a^{r/2} + 1)$ has a factor in common with N. (GCD is easy to calculate.)

Example: $N = 15$ and $a = 2$.

$2^x \bmod 15 = 1, 2, 4, 8, 16 \to 1, 32 \to 2, \ldots \ \Rightarrow\ r = 4, r/2 = 2.$

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Periodic patterns are easily detected by Fourier Transform, which is a familiar unitary operation in quantum theory.

Quantum Fourier Transform

$$\sum_x f(x)|x\rangle = \sum_y \left(\frac{1}{\sqrt{N}} \sum_x e^{2\pi i xy/N} f(x) \right) |y\rangle$$

Let $N = 2^n$, and use the same tricks as in FFT.

In binary notation, $x = x_{n-1} \cdot 2^{n-1} + \ldots + x_1 \cdot 2 + x_0$.

$\mathrm{frac}(\frac{xy}{N}) = y_{n-1}(.x_0) + y_{n-2}(.x_1 x_0) + \ldots + y_0(.x_{n-1} \ldots x_0)$.

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Unitary rotation of QFT: $|x\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_y e^{2\pi ixy/N} |y\rangle$

$$= \frac{\left(|0\rangle + e^{2\pi i(.x_0)}|1\rangle\right)}{\sqrt{2}} \frac{\left(|0\rangle + e^{2\pi i(.x_1 x_0)}|1\rangle\right)}{\sqrt{2}} \cdots \frac{\left(|0\rangle + e^{2\pi i(.x_{n-1} \ldots x_0)}|1\rangle\right)}{\sqrt{2}}$$

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Fourier Transform is a multiplication by an $N \times N$ matrix.
FFT factorisation reduces the operations to $O(N \log N)$.
QFT parallelism cuts down the operations to $O((\log N)^2)$.

Quantum random walk

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Quantum computers can explore multiple evolutionary branches of an algorithm—in a single attempt—by using clever superpositions of states. (Coin is unnecessary.)

Quantum diffusion

Random walks represent a diffusion process.

Classical diffusion operator is the Laplacian: $\frac{\partial P}{\partial t} = \nabla^2 P$.

A spatial mode with wave vector $\vec{k}$ evolves as $\exp(-E(\vec{k})t)$, with $E(\vec{k}) \propto |\vec{k}|^2$. The slowest propagating modes (small $\vec{k}$) produce the characteristic Brownian motion signature:

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But there is an alternative. Relativistic Dirac equation uses the diffusion operator $\vec{\alpha} \cdot \vec{\nabla}$ (α_i are anticommuting objects, e.g. Pauli matrices), with $E(\vec{k}) \propto |\vec{k}|$ and the signature:

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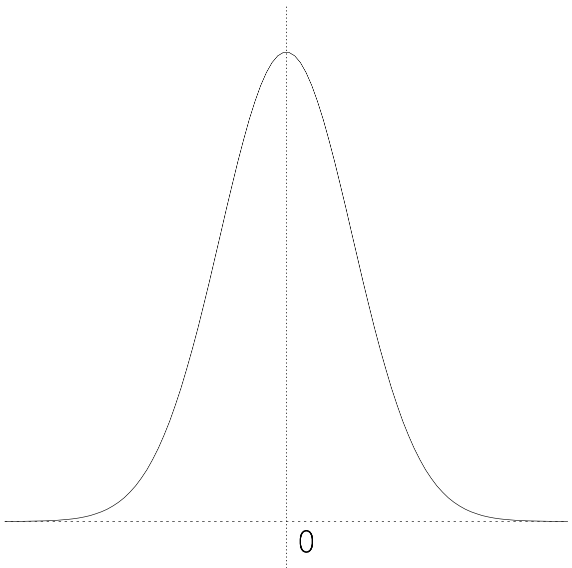
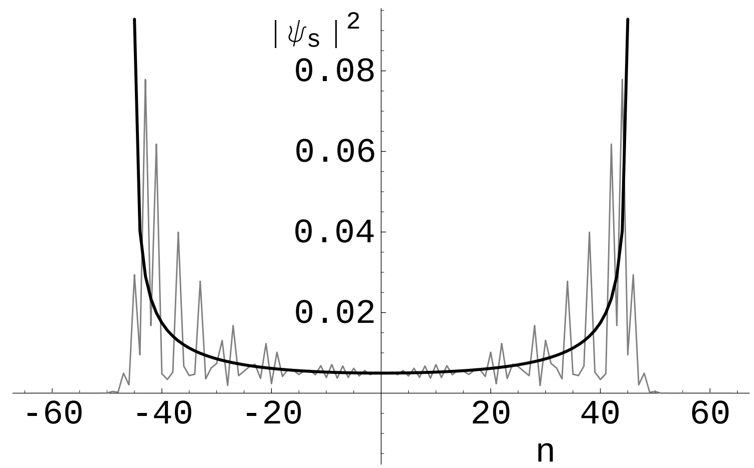
Any NP-complete problem speeds up at least quadratically.

Random walk on a line





$$P(x,t) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/2t}$$

$$\int P(x,t)dx = 1$$

$$\int |x| \cdot P(x,t)dx = \sqrt{\frac{2t}{\pi}}$$

$$\int x^2 P(x,t)dx = t$$

$$|\psi_s|^2 = \frac{4t^2}{\pi\sqrt{4t^2-2n^2}\ (4t^2-n^2)}$$

$$\int_{n=-\sqrt{2}t}^{\sqrt{2}t} |\psi_s|^2 dn = 1$$

$$\int_{n=-\sqrt{2}t}^{\sqrt{2}t} |n| \cdot |\psi_s|^2 dn = t$$

$$\int_{n=-\sqrt{2}t}^{\sqrt{2}t} n^2 |\psi_s|^2 dn = 2(2-\sqrt{2})t^2$$

Probability distributions for symmetric random walks:

Left: The classical one is a Gaussian.

Right: The quantum one is double peaked.

Grover's Quantum Database Search

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$|\langle i|s\rangle|^2 = 1/N, \quad |\langle i|t\rangle|^2 = \delta_{it}.$

Operators: Reflections along $|t\rangle$ and $|s\rangle$ directions.

$U_t = 1 - 2|t\rangle\langle t|$ (Potential energy attraction)

$U_s = 1 - 2|s\rangle\langle s|$ (Kinetic energy diffusion)

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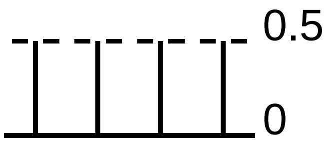
The algorithm is optimal, evolving the starting state $|s\rangle$ to the target state $|t\rangle$ along the shortest geodesic route.

Compared to its $O(\sqrt{N})$ scaling, any algorithm based on Boolean logic needs $O(N)$ oracle calls.

An example

The key feature of the algorithm is wave dynamics, and not entanglement.

Using a single oracle call, the algorithm identifies 1 out of 4 items in the database. In contrast, a Boolean algorithm identifies only 1 out of 2 items.

	Amplitudes	Algorithmic Steps	Physical Implementation
(1)		Uniform distribution	Equilibrium configuration

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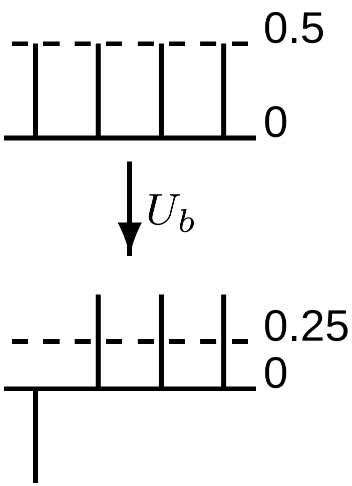

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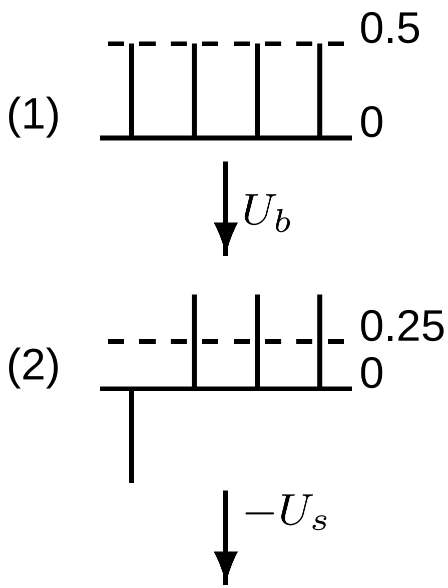

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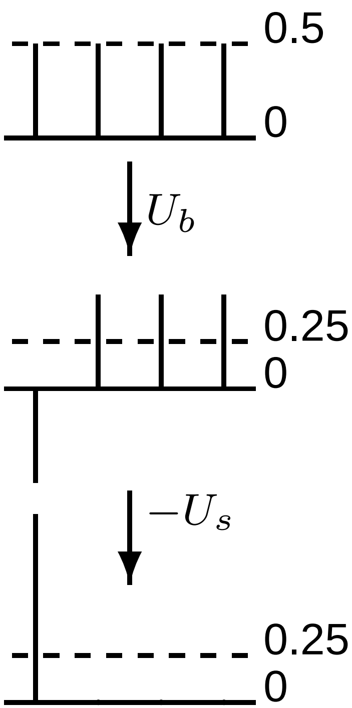

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(3)		Desired state reached	Opposite end of oscillation
(4)	Projection	Algorithm is stopped	Measurement

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A mechanical model

Grover's algorithm is an amplitude amplification process.
A system of coupled wave modes can execute it, provided
(1) Superposition of modes maintains phase coherence.
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Consider N identical coupled harmonic oscillators. Identical coupling between them is arranged by attaching them to a big oscillator through the centre-of-mass mode.
Elastic reflection of an oscillator implements the binary oracle in momentum space. Evolution by half an oscillation period implements the reflection about average operation.

Possible uses

Decoherence of quantum behaviour is extremely fast, but vibrational systems with small damping can be made easily.

Focusing of energy:

Concentration of total energy of a coupled oscillator system into a specific oscillator can have potential applications in processes that are highly sensitive to energy availability.

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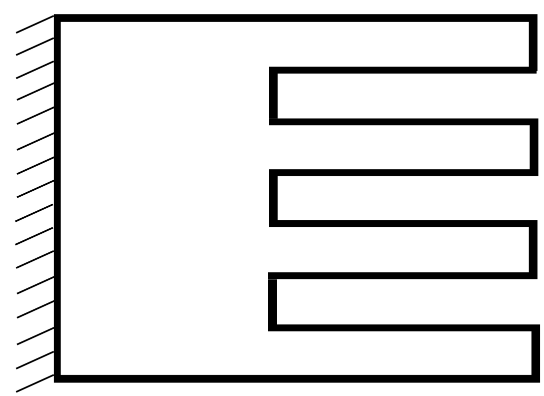
Nanomechanical devices: A coupled oscillator system can provide efficient focusing of energy at a specific location, when one cannot directly control the component concerned.

For example, a comb-shaped cantilever beam can be used as a selective switch.



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Dispersal of energy:

The algorithm is fully reversible, and running it backwards, i.e. $(-U_t U_s)^Q|t\rangle = |s\rangle$, distributes large initial energy in one of the oscillators equally among its partners.

Catalysis: There exist many processes that need crossing of an energy threshold for completion. Their rates are typically governed by the Boltzmann factor for the energy barrier, $\exp(-E_{\rm barrier}/kT)$. Energy amplification can speed up the rates of such processes by large factors.

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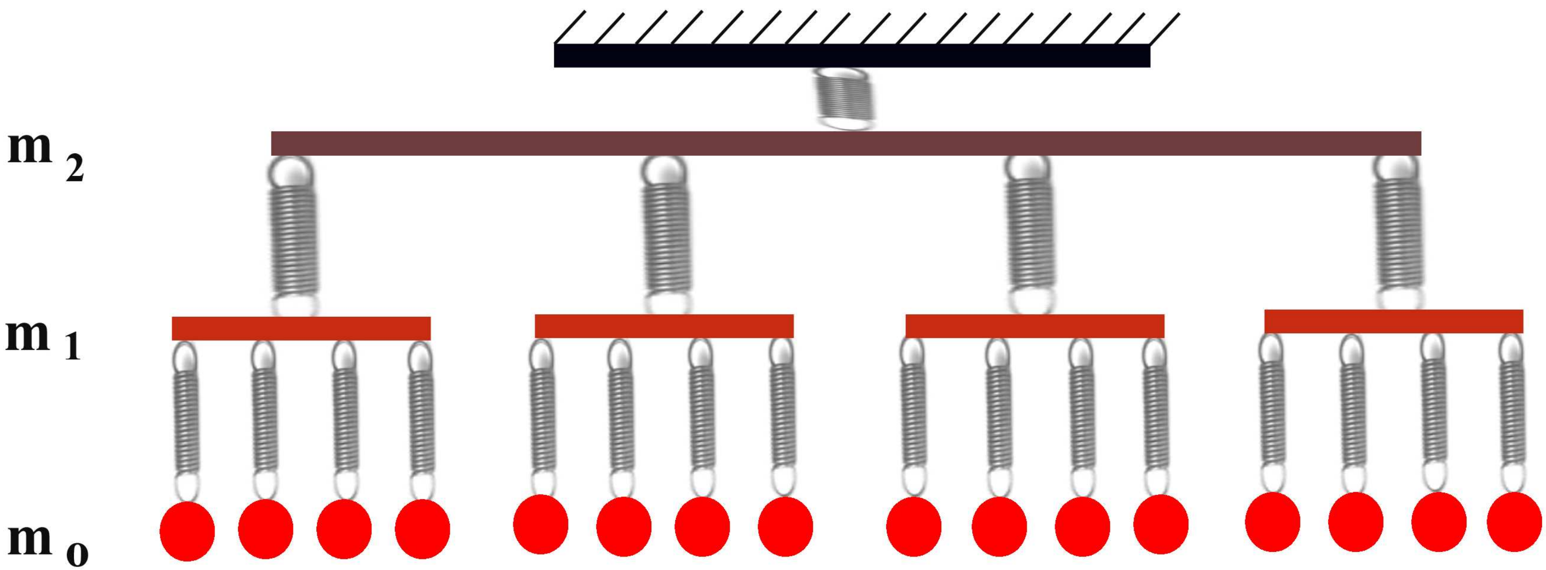
Shock absorbers and vibrational isolation: Instead of damping a single perturbed oscillator, it is much more efficient to disperse the energy into several oscillators while damping them together.

A hierarchical system of oscillators—four small ones coupled to a big one at every level with appropriate mass, spring and damping parameters—can provide a practical realisation of this idea.



(The initial impulse is taken to be a local disturbance, which subsequently spreads out.)

Genetic languages

1. What is the information processing task carried out by the genetic code?
   Assembling molecules by picking up components from an unsorted database.
2. What is the optimal way of carrying out this task?
   Lov Grover's quantum search algorithm.
   (Requires wave dynamics.)
3. What is the signature of this algorithm?

$$(2Q+1)\sin^{-1}\frac{1}{\sqrt{N}} = \frac{\pi}{2} \implies \begin{cases} Q=1, & \text{N=4} \\ Q=2, & \text{N=10.5} \\ Q=3, & \text{N=20.2} \end{cases}$$

Lessons from Molecular Biology

Molecular biology is a nanotechnology that works—it has worked for billions of years and in an amazing variety of circumstances. Darwinian evolution has taken its basic processes to their highly optimised and essentially universal forms. By looking at them as information processing tasks, we can analyse what has been optimised and how.

Telltale signatures of quantum effects and wave dynamics show up in several instances. Examples are enzyme catalysis, photosynthesis and structure of genetic languages. Obviously, a fundamental understanding of molecular biology would have a lot to say about what we may design or convert ourselves into.

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Enzyme Catalysis: Reaction rate enhancements range from 10^6 to 10^{12}.
Chemical industry reaches $10^3 - 10^6$.

Photosynthesis: Coherent oscillations last for longer than 500fs.
No coherence longer than 100fs was expected.

Genetic languages: No. of letters in the alphabet fit Grover's algorithm.
The languages are considered frozen accident.

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