Information Theory Meets Quantum Physics

The magic of wave dynamics

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The availability of different physical interactions makes it possible to design different types of computers.

Communication is the special case where processing is limited to coding and decoding.

Designing a Computer

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Efficiency of information processing depends on both the available hardware and the possible software.

Software: What is the task? What is the algorithm?

Hardware: How are the operations implemented?

Technical Terms

Data: They describe a particular realisation of the physical system, amongst its many possible states.

Information: It is the abstract mathematical property obtained by detaching all the physical characteristics from data.

Knowledge: It is obtained by adding a sense of purpose to the abstract information.

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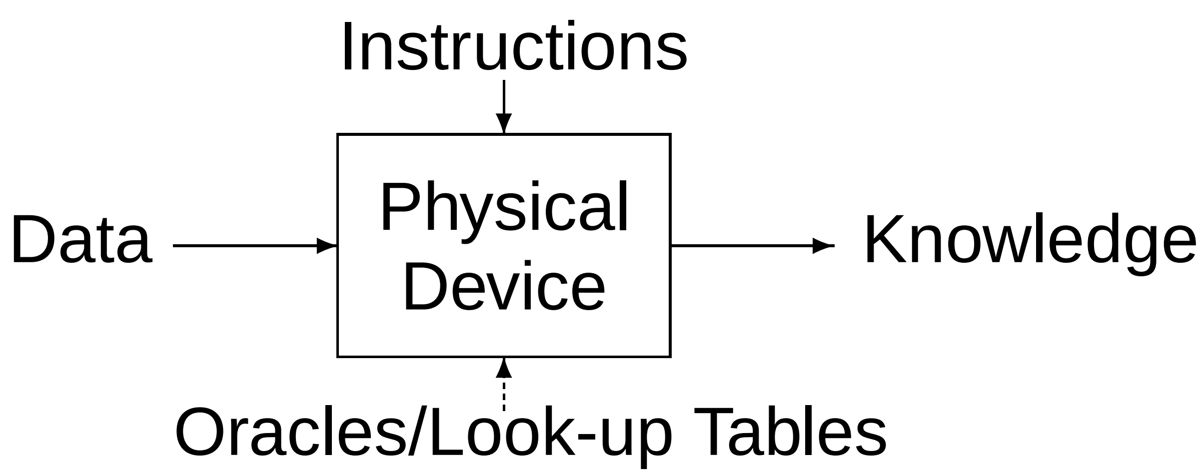
Information = Data - Physical Realisation
Knowledge = Information + Interpretation



Importance of Physics

1. Abstract information can be manipulated with precise mathematical rules, without going into nitty-gritty of its origin or meaning.
2. The manipulations can only be implemented using physical devices.
3. The interpretation of a language has to be established through physical properties.

Physical hardware properties govern the expression and the grammar of the language.

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Abstract information theory does not tell us what physical realisation would be appropriate for a particular message, nor does it tell us the best way of implementing a computational task.

These choices have to be made by analysing the type (and not the amount) of information, and checking how that can be mapped to the available physical resources.

An Example from Biology

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Direction: Plant releases millions of fragrant molecules. Insect detects gradients (parallax) with multiple receptors.
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Mobile phones convey existence (followed by two-way communication), but direction and distance are ignored.

Minimal Language

The language with the smallest set of building blocks (for a given task) is unique in the optimisation procedure.

- Largest tolerance against errors.

(Discrete variables are spread as far apart as possible in the available range of physical hardware properties.)

- Smallest instruction set.

(Number of possible transformations is limited.)

- High density of packing and quick operations.

(These more than make up for the increased depth of computation.)

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Boolean algebra provides the minimal classical language for encoding information as 1-dimensional sequences.

General Computational Framework

Generalise the concept of a language, from a "sequence of letters" to a "collection of building blocks".

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Generalise the concept of a language, from a "sequence of letters" to a "collection of building blocks".

Collections: The building blocks can be arranged in the space-time in many different ways.
Building blocks: Physical properties (generically encoded using groups) express the meaning of the building blocks.
Processing: Allowed changes in the properties of building blocks exhaust the possible manipulations of information.

Group representations fix the structure of the language, and group transformations provide the rules for processing information.

. . . contd.

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The dimension of a group is the number of its generators. In the dual description, the minimal building block set is the d-dim fundamental representation and the 1-dim identity representation.

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In the smallest discrete realisation, the entire group is replaced by a single simplex. A collection of simplices can then represent any quantity to the desired precision (e.g. the place value system for numbers).

Types of Collections

$0 - dim$: Multiple signals at the same point in space and time, i.e. superposition. Different states of internal degrees of freedom encode different signals. Only one signal can be extracted at a time.

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$2 - dim$: Combination of multiple ordered sequences. Information can reside in correlations amongst sequences without being present in any individual sequence.
Examples: Eyes and ears, Space-time codes,
Parallax and gradient detection.

Types of Collections (contd.)

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Particle features: Countability, Order, Number density, Shape, Structure.
Wave features: Superposition, Differential analysis (parallax), Interference (multiple paths).

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$2 - dim$: The simplex is a triangle. Triangulation (or its dual hexagonal form) is useful for discrete description of arbitrary surfaces. Graphene structure may become useful in atomic scale lithography.

Types of Building Blocks (contd.)

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$SU(2)$: Description of quantum bits is based on this group with three generators. Arbitrary states of a qubit (including mixed states) can be fully described using a density matrix, which is a linear combination of the four operators $\{1, \sigma_x, \sigma_y, \sigma_z\}$. (Four real numbers $\equiv$ Two complex numbers)

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Larger groups have been used in error correcting codes and cryptography, but not for processing information.

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$d > 1$: In higher dimensions, addition generalises to translation, and multiplication to scale transformation.
But rotations appear as well (non-commutative for $d > 2$).
Algebra generated by lattice transformations is much more powerful than common arithmetic.

More and more group operations become possible with increasing dimensionality.

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Classical (particle dynamics):
Discrete Boolean logic is implemented using digital circuits.

Wave (classical wave dynamics):
Analog variables can superpose, interfere, disperse etc., and are convenient for differentiation and integration.

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Quantum (particle+wave dynamics):
Unitary evolution is implemented using quantum states. Combination of particle and wave properties produces unusual correlations called entanglement.

Basics

The simplest quantum system is a qubit, with two basis vectors $|0\rangle$ and $|1\rangle$ (e.g. $|\uparrow\rangle$ and $|\downarrow\rangle$ for an electron spin).
A generic qubit state is a 2-dim complex unit vector.

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1.$$

A quantum register is an ordered string of n qubits.
It is a complex unit vector in the 2^n-dim Hilbert space.

$$|x\rangle = \sum_{i_1,i_2\ldots i_n=0}^{1} c_{i_1 i_2 \ldots i_n} |x_{i_1}\rangle |x_{i_2}\rangle \ldots |x_{i_n}\rangle, \sum_{i_1,i_2\ldots i_n=0}^{1} |c_{i_1 i_2 \ldots i_n}|^2 = 1.$$

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A generic instruction is a rotation of the quantum state vector in the Hilbert space. It is a unitary transformation that is deterministic and fully reversible.

A measurement is a projection. In the computational basis, it yields the state $|x_{i_1}\rangle|x_{i_2}\rangle \ldots |x_{i_n}\rangle$ with probability $|c_{i_1 i_2 \ldots i_n}|^2$.
This operation is probabilistic and irreversible.

Communication Tasks

Singlet Bell State: $|\psi_-\rangle = (|0\rangle|0\rangle - |1\rangle|1\rangle)/\sqrt{2}$

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Dense coding: A Bell state exists spread over two locations. One of the four operators $\{I, X, Z, XZ\}$ is applied to the half Bell state at one end, and the qubit is sent to the other end. Joint Bell basis measurement of the two qubits determines which of the four operators was applied.

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Teleportation: A Bell state exists spread over two locations. The unknown state to be teleported from one end is jointly measured with the half Bell state in Bell basis. The two-bit measurement result sent to the other end recreates the unknown state from the other half of the Bell state.

$$|\alpha\rangle|\psi_-\rangle = (|\psi_-\rangle|\alpha\rangle + |\phi_-\rangle X|\alpha\rangle + |\psi_+\rangle Z|\alpha\rangle + |\phi_+\rangle ZX|\alpha\rangle)/2$$

Communication Tasks (contd.)

Superadditivity: More information can be sent through an n-product channel than n times the amount that can be sent through a single use of a channel. Quantum correlations between the signals provide the extra channel capacity.

Zero capacity channels can be combined to obtain a nonzero communication rate! (Signal and noise can be separated when they have different correlation scales.)

Perfect entanglement between inputs to the channels can even eliminate the noise.

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Classical vs. Quantum:
Shannon entropy is generalised to von Neumann entropy.

$$H(\{p_i\}) = -\sum_i p_i \log p_i \longrightarrow S(\rho) = -\mathrm{Tr}(\rho \log \rho).$$

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Shannon's theorems:

- Data compression is analogous to the classical case.
- Channel capacity is not analogous to the classical case.

Input and output are always classical in problems relevent to us.

Power of One Qubit

The DQC1 model has one control qubit, n others in a fully mixed state, and a unitary operation between them.

$$\rho_i = \frac{1}{2}(I + \alpha X) \otimes \frac{1}{2^n} I^{\otimes n} \xrightarrow{Control-U_n} \rho_f = \frac{1}{2^{n+1}} \begin{pmatrix} I^{\otimes n} & \alpha U_n^\dagger \\ \alpha U_n & I^{\otimes n} \end{pmatrix}$$

Measurement of the control qubit gives:

$$\langle X \rangle = \alpha \ \mathrm{Re}(\mathrm{Tr} U_n)/2^n, \ \langle Y \rangle = \alpha \ \mathrm{Im}(\mathrm{Tr} U_n)/2^n$$

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There exist $2^n \times 2^n$ unitary operations U_n that can be implemented using $poly(n)$ one- and two-qubit operations, but whose trace cannot be computed with a known $poly(n)$ classical algorithm. That makes the DQC1 model more powerful than its classical counterpart.

There is no entanglement between the control qubit and the mixed state in this example.

Database Search

Classical:

Binary tree search is the optimal classical algorithm.
A sorted database of N items can be searched using $\log_2 N$ binary questions.
An unsorted database of N items can be searched using $N/2$ binary questions with memory, and using N binary questions without memory.

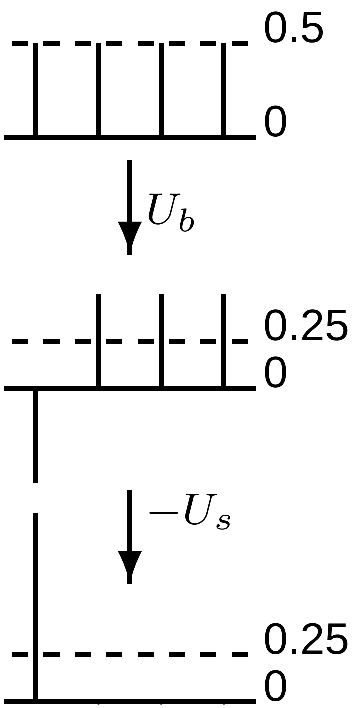
Quantum/Wave:

Wave mechanics works with amplitudes and not with probabilities. Superposition of amplitudes can yield constructive as well as destructive interference.
Optimal search solutions differ from the classical ones.

Grover's algorithm: An unsorted database of N items can be optimally searched using $(\pi/4)\sqrt{N}$ binary questions.

Grover's Database Search

The steps of the algorithm for the simplest case of 4 items in the database. Let the first item be desired by the oracle.



	Amplitudes	Algorithmic Steps	Physical Implementation
(1)		Uniform distribution	Equilibrium configuration
	U_b	Quantum oracle	Binary question
(2)		Amplitude of desired state flipped in sign	Sudden perturbation
	$-U_s$	Reflection about average	Overrelaxation
(3)		Desired state reached	Opposite end of oscillation
(4)	Projection	Algorithm is stopped	Measurement

(Dashed line denotes the average amplitude.)

One binary question unambiguously distinguishes 4 items.

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All my papers are available at http://arXiv.org/

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